Effective Field Theories for Heavy Quarks

Part III: Heavy Quark Expansion and Soft Collinear Effective Theory

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What is still missing?

- Discussion of inclusive decays:
  $B \to X_c \ell \bar{\nu}_\ell$, Lifetimes Mixing ...
  Heavy Quark Expansion
- Decays heavy hadrons into light quarks:
  Very Energetic light degrees of freedom: $E \sim m_b$
  Soft Collinear Effective Theory

Literature for Part III

- A Manohar, M. Wise,
  Heavy Quark Physics, Cambridge UP
- T. Mannel, Springer Tracts 2005
- SCET Paper series by Bauer, Pirjol, Stewart
- SCET Paper series by Beneke, Feldmann

Contents Part III

Heavy Quark Expansion for Inclusive Decays: Set up

Heavy Quark Expansion = Operator Product Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}_{eff}^\dagger(0) | B(v) \rangle$$

$$= 2 \, \text{Im} \int d^4x \, \langle B(v) | T\{\mathcal{H}_{eff}(x) \mathcal{H}_{eff}^\dagger(0)\} | B(v) \rangle$$

$$= 2 \, \text{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}_{eff}^\dagger(0)\} | B(v) \rangle$$

- Last step: $p_b = m_b v + k$,
  Expansion in the residual momentum k

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\longrightarrow$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\longrightarrow$ Perturbation theory!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle$$
$$2M_H\mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle$$
$$2M_H\rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle$$

ρ_D: Darwin Term and ρ_{LS}: Chromomagnetic moment

- Γ_4: Nine more Parameters At $\mathcal{O}(1/m^4)$
  Dimension 7 matrix elements = four derivatives
  In terms of physical quantities:

Spin-independent

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

Spin-dependent

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

- Beyond this there is a real proliferation of parameters!
- This becomes useless unless one has a way to compute the matrix elemets

Remarks:

- All these parameters are of the order $(\Lambda_{\rm QCD}/m_b)^n$ with appropriate n.
- For $\Lambda_{\rm QCD} \sim 500 MeV$ and $m_b = 4.6$ GeV: $\Lambda_{\rm QCD}/m_b = 0.11$
- The leading nonperturbative Corrections are of order $(\Lambda_{\rm QCD}/m_b)^2 = 1\%$
- Nonperturbative corrections are expected to be small!
- This is an embarrasment:
  - Significant lifetime difference between D^0 and D^+
  - Even larger lifetime differences between D^0 and Λ_c
  - ...

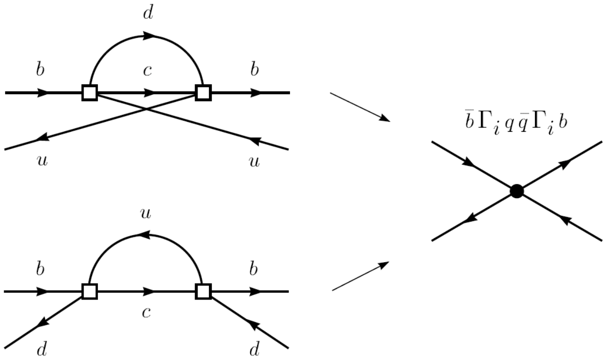
A short look at lifetimes

- Calculate the Coefficients ...
- Up to Isospin breaking $\mu_\pi(B^+) = \mu_\pi(B^0)$
- $\to$ Meson lifetime differences are order $(\Lambda_{\rm QCD}/m_b)^3$
- Expectation:

$$\frac{\tau(M^-)}{\tau(M^0)} = 1 + O(1/m_Q^3)\,,$$
$$\frac{\tau(M_s)}{\tau(M^0)} = 1 + O(1/m_Q^3)\,,$$
$$\frac{\tau(\Lambda_Q)}{\tau(M^0)} = 1 + O(1/m_Q^2)$$

- This does not look good for charm, however ...

Look at the diagrams:



- These diagrams have one less loop compared to the leading terms
- ... yields a relative factor $16\pi^2 \sim 158$

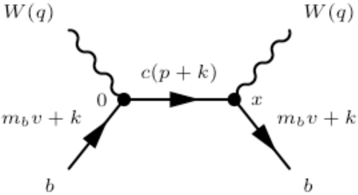
Hence

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + \frac{1}{m_b^3}\left[a_0 + \frac{1}{m_b}a_1 + \cdots\right] + \frac{16\pi^2}{m_b^3}\left[b_0 + \frac{1}{m_b}b_1 + \cdots\right] \quad ,$$

- Relative enhancement of the $1/m_Q^3$ terms
- Satisfactory explanation of the lifetme patterns

Spectra of Inclusive Decays

- Calculation proceeds in the same way

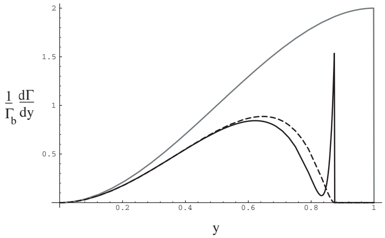


- Denominator of the charm propagator:

$$(m_b v + k - q)^2 - m_c^2 = m_b^2 - m_c^2 + 2m_b(vq) + q^2 + \cdots$$

- Sbould be large against Λ_{QCD}
- ... this cannot be in all phase space

Lepton Energy Epectrum



- Endpoint region: $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho)\left[2 + \frac{\lambda_1}{(m_b(1-y))^2}\left(\frac{\rho}{1-y}\right)^2\left\{3 - 4\frac{\rho}{1-y}\right\}\right]$$

- Break-down of the HQE in the endpoint region
- ... but reliable calculation for *moments* of specta

Moments of Spectra

- Charged lepton energy spectrum
- Hadronic invariant mass spectrum

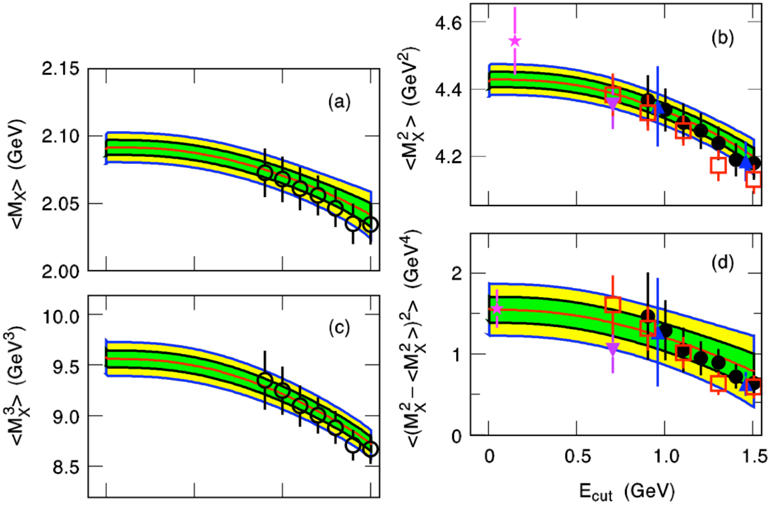
$$\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X \, M_X^n \int_{E_{\rm cut}} dE_\ell \, \frac{d^2\Gamma}{dM_x \, dE_\ell}$$
$$\langle E_\ell^n \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\rm cut}} dE_\ell \, E_\ell^n \frac{d^2\Gamma}{dM_x \, dE_\ell}$$

- Moment are sensitive to higher orders of HQE:

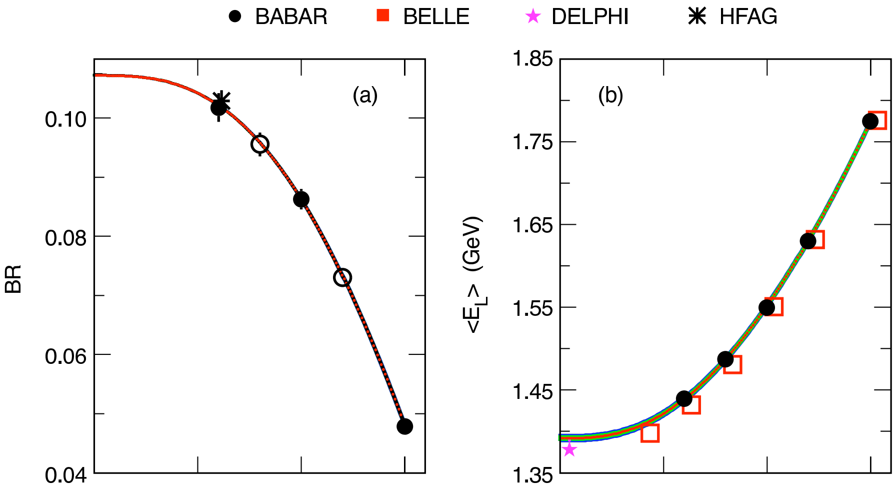
$$\langle M_X^n \rangle \sim \mathcal{O}\left[\left(\frac{1}{m_b}\right)^n\right]$$

- May be used to extract the HQE parameters!

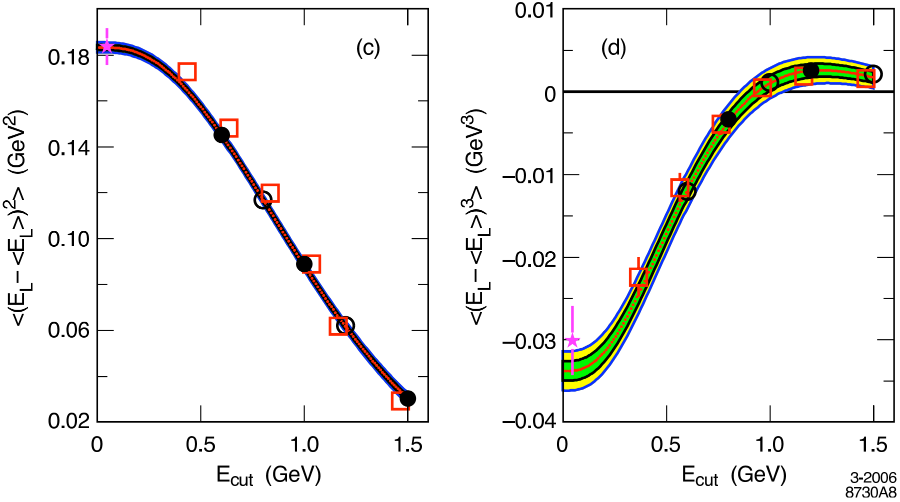
Hadronic Invariant Mass Moments (Buchmüller, Flächer)



Lepton Energy Moments I (Buchmüller, Flächer)



Lepton Energy Moments II (Buchmüller, Flächer)



$$\boxed{V_{cb,incl} = (41.54 \pm 0.44 \pm 0.59_{HQE}) \times 10^{-3}}$$

(PDG 2010)

Heavy to light inclusive: $B \to X_u \ell \bar{\nu}$

- Formulae also apply for $b \to u$, taking the limit $m_c \to 0$ and $V_{cb} \to V_{ub}$
- Useless for the extraction of V_{ub}!
  Due to the $b \to c$ background.
- Phase space cuts ruin the stadard OPE
  (at last for most of the proposed methods)
- Partial Resumation: Nonperturbative input will be a "shape functon"
- Simple way to explain this: $BtoX_s \gamma$

Photon Spectrum in $B \to X_s \gamma$

- The photon spectrum becomes (with $x = 2E_\gamma / m_b$)

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb*}|^2 |C_7|^2$$
$$\left(\delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \delta'(1-x) + \frac{\lambda_1}{6m_b^2} \delta''(1-x) + \cdots \right)$$

- General Structure at tree level (no real gluons)

$$\frac{d\Gamma}{dx} = \Gamma_0 \left[\sum_i a_i \left(\frac{1}{m_b} \right)^i \delta^{(i)}(1-x) + \mathcal{O}((1/m_b)^{i+1} \delta^{(i)}(1-x) \right]$$

- The same happens with the other spectra !
- Interpretation: The expansion parameter is not $1/m$, rather it is $1/[m(1-x)]$: Expansion breaks down in the Endpoint $x \sim 1$.
- Comparison with experiment: Moments:

$$M_n = \int_0^1 dx \, (1-x)^n \frac{d\Gamma}{dx}$$

- $M_0 = \Gamma \qquad M_1 = -\frac{\lambda_1 + 3\lambda_2}{2m_b^2} \qquad M_2 = \frac{\lambda_1}{6m_b^2} \qquad \cdots$
- Power counting for the moments: $M_n = \mathcal{O}(1/m^n)$

Resummation and shape function

- Leading terms can be resummed into a shape function:

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 f(1-x)$$

where $2M_B f(\omega) = \langle B | \bar{Q}_v \delta(\omega + n \cdot (iD)) Q_v | B \rangle$
$n \cdot (iD)$: light-cone component of residual momentum.

- Fourier transform of a (light-like) Wilson line:

$$f(\omega) = \int \frac{dt}{2\pi} \langle B(v) | \bar{Q}_v(0) \mathcal{P} \exp\left(-i \int_0^t ds\, n \cdot A(n \cdot s)\right) Q_v(n \cdot t) | B(v) \rangle$$

Shape Function: Kinematics

Where in phase space is the shape function relevant?

$$p_B = M_B v = q + p \text{ with } q^2 = 0$$

- Light cone vectors n and $\bar{n}$ with $n \cdot \bar{n} = 2$
- One of which is $q = \frac{1}{2}(n \cdot q)\bar{n}$
- The second is from $v = \frac{1}{2}(n + \bar{n})$
- $m_b v - q = \frac{1}{2}[m_b\, n + (m_b - n \cdot q)\bar{n}]$
- Shape function region:
  - Final state invariant mass $(p_B - q)^2 \sim \mathcal{O}(\Lambda_{QCD} m_b)$
  - Hadronic energy $M_B - v \cdot q \sim \mathcal{O}(m_b)$

Examples

- $B \to X_s \gamma$
  - $q_\gamma^2 = 0$ and $q = E_\gamma \bar{n}$
  - $E_\gamma = \frac{M_B}{2} - \mathcal{O}(\Lambda_{\rm QCD})$

  $\to$ Endpoint of the Photon Energy Spectrum ($2E_\gamma \to M_B$)
- $B \to X_u \ell \bar{\nu}_\ell$
  - $m_\ell = 0$ and $q = E_\ell \bar{n}$
  - $E_\ell = \frac{M_B}{2} - \mathcal{O}(\Lambda_{\rm QCD})$

  $\to$ Endpoint of the Lepton Energy Spectrum ($2E_\ell \to M_B$)
- The same non-perturbative input !

Simple (tree level) derivation

We start from:

$$\int d^4x e^{-i(m_b v - q)\cdot x} \langle B(v)| T\{ b_v(x) \Gamma q(x) \bar{q}(0) \bar{\Gamma} b_v(0)\} |B(v)\rangle$$

and contract the light quark

$$\begin{aligned} &\langle B(v)| T\{ b_v(x) \Gamma q(x) \bar{q}(0) \bar{\Gamma} b_v(0)\} |B(v)\rangle \\ &= \langle B(v)| h_v(x) \Gamma \, \langle 0| T\{ q(x) \bar{q}(0)\} |0\rangle \, \bar{\Gamma} b_v(0) |B(v)\rangle \end{aligned}$$

expand the light (massless) Propagator (in external field)

$$\langle 0| T\{ q(x) \bar{q}(0)\} |0\rangle \stackrel{F.T.}{=} \frac{m_b \not{v} - \not{q} + i\not{D}}{(m_b - q)^2 + 2(m_b - q)\cdot(iD) + (i\not{D})^2 + i\epsilon}$$

- Numerator: $m_b \not{v} - \not{q} + i\not{D} = \frac{m_b}{2} \not{n} + \mathcal{O}(\Lambda_{\rm QCD})$
- Denominator:

$$(m_b - q)^2 + 2(m_b - q) \cdot (iD) + (i\not{D})^2$$
$$= m_b[m_b - n \cdot q + n \cdot (iD) + i\epsilon] + \mathcal{O}(\Lambda^2_{\rm QCD})$$

- Collect everything:

$$\int d^4x e^{-i(m_b v - q)\cdot x} \langle B(v)| T\{b_v(x) \Gamma q(x) \bar{q}(0) \bar{\Gamma} b_v(0)\} |B(v)\rangle$$
$$= \langle B(v)| h_v \Gamma \frac{1}{2} \not{n} \bar{\Gamma} \left(\frac{1}{m_b - n \cdot q + n \cdot (iD) + i\epsilon} \right) h_v |B(v)\rangle$$
$$+ \cdots$$

$\rightarrow$ Leads to a convolution with the shape function

Why and where SCET?

- Power Counting: (p_{fin}: light final state)

$$p_{\rm fin} = \frac{1}{2}(n \cdot p_{\rm fin})\bar{n} \text{ and } v = \frac{1}{2}(n + \bar{n})$$

- Momentum of a light quark in such a meson:

$$p_{\rm light} = \frac{1}{2}[(n \cdot p_{\rm light})\bar{n} + (\bar{n} \cdot p_{\rm light})n] + p_{\rm light}^{\perp}$$

- The light quark invariant mass (or virtuality) is assumed to be

$$p_{\rm light}^2 = (n \cdot p_{\rm light})(\bar{n} \cdot p_{\rm light}) + (p_{\rm light}^{\perp})^2 \sim \lambda^2 m_b^2$$

- Quark momentum has to scale as

$$(n \cdot p_{\rm light}) \sim m_b \quad (\bar{n} \cdot p_{\rm light}) \sim \lambda^2 m_b \qquad p_{\rm light}^{\perp} \sim \lambda m_b$$

The Lagrangian of SCET

- Analogous to HQET:
  - Pick the $\mathcal{O}(\lambda)$ and $\mathcal{O}(\lambda^2)$ of the quark momentum:

$$Q = \frac{1}{2}(n \cdot p_{\text{light}})\bar{n} + p_{\text{light}}^{\perp}$$

  - Rephase the quark fields: $\psi(x) = \sum_Q e^{-iQ\cdot x}\psi_{n,Q}(x)$
  - Project out the “large” and “small” components using

$$\xi_{n,Q}(x) = \mathcal{P}\psi_{n,Q}(x) \text{ with } \mathcal{P} = \frac{1}{4}\not{n}\not{\bar{n}} \quad \text{and}$$

$$\eta_{n,Q}(x) = \mathcal{Q}\psi_{n,Q}(x) \text{ with } \mathcal{Q} = \frac{1}{4}\not{\bar{n}}\not{n}$$

- The Lagrangian becomes

$$\mathcal{L} = \sum_{p,p'} e^{-ix(p-p')} \Big[\bar{\xi}_{n,p'} \frac{\not{n}}{2} (in \cdot D) \xi_{n,p} + \bar{\eta}_{n,p'} \frac{\not{n}}{2} [\bar{n} \cdot p + (i\bar{n} \cdot D)] \eta_{n,p}$$
$$+ \bar{\xi}_{n,p'} \left(\not{p}_\perp + i\not{D}_\perp \right) \eta_{n,p} + \bar{\eta}_{n,p'} \left(\not{p}_\perp + i\not{D}_\perp \right) \xi_{n,p} \Big]$$

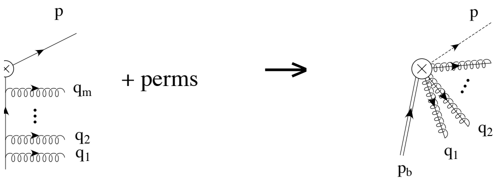
- Integrate out the “heavy”degree of freedom η
  $\rightarrow$ use the equations of motion for η

$$\mathcal{L} = \sum_{p,p'} e^{-ix(p-p')} \Big[\bar{\xi}_{n,p'} \frac{\not{n}}{2} (in \cdot D) \xi_{n,p} +$$
$$\bar{\xi}_{n,p'} \left(\not{p}_\perp + i\not{D}_\perp \right) \frac{1}{\bar{n} \cdot p + (i\bar{n} \cdot D)} \eta_{n,p} \left(\not{p}_\perp + i\not{D}_\perp \right) \frac{\not{n}}{2} \xi_{n,p} \Big]$$

- Similarly for Gluons ...

Matching of Heavy-Light Currents

- $\bar{q}\Gamma Q \longrightarrow \bar{\xi}_{n,p'}\mathcal{W}(0)\Gamma h_v$: $\mathcal{W}$: Resummation of collinear gluons



- SCET contains non-localities, expressed in terms of Wilson-lines

$$\mathcal{W}(x) = \mathcal{P}\exp\left(-i\int_{-\infty}^{0} ds\ \bar{n}\cdot A_c(x+s\bar{n})\right)$$

Radiative corrections in heavy-to-light inclusive decays

- Matching proceeds in two steps:

(a) Matching QCD $\to$ SCET

$$T\left[\bar{Q}(x)\Gamma q(x)\bar{q}(0)\bar{\Gamma}Q(0)\right] \longrightarrow$$
$$T\left[\bar{h}_v(x)\Gamma\mathcal{W}^\dagger(x)\xi_{n,p'}(x)\bar{\xi}_{n,p'}\mathcal{W}(0)\bar{\Gamma}h_v(0)\right]$$

(b) Matching SCET $\to$ Nonlocal operators

$$T\left[\bar{h}_v(x)\Gamma\mathcal{W}^\dagger(x)\xi_{n,p'}(x)\bar{\xi}_{n,p'}\mathcal{W}(0)\bar{\Gamma}h_v(0)\right] \longrightarrow$$
$$\int d\omega\, C(\omega)h_v\delta(\omega + in\cdot D)h_v$$

Leads to a convolution of the structure

$$d\Gamma = H * J * f$$

- H: Hard Contibution
- J: Jet Function:
  collinear modes (contains the Sudakov Logs.)
- f: Shape Function:
  soft contributions, nonperturbative

This has numerous applications:

- Semileptonic Decays: Precision determination of V_{ub}
- Precise predictions for $B \to X_s \gamma$
- Applies also for exclusive (non-leptonic) decays:
  QCD Factorization, SCET ...
  ongoing debate with the PQCD group
  seems to have problems to explain some of the data

Summary of Part III

- Inclusive decays can be described also in a HQE
  Make use of the optical theorem
- Standard OPE has a certain set of non-perturbative parameters: $\mu_\pi, \; \mu_G, \; , \rho_D, \; \rho_{LS} \cdots$
- In the standard approach: Light degrees of freedom have to be "slow": $(vp)^2 \sim p^2 \sim \Lambda_{\rm QCD}^2$
- Other extreme can also be handled:
  $vp \sim m_b$ but $p^2 \sim \Lambda_{\rm QCD}^2$
  energetic light degrees of freedom: SCET

Summary of “Everything”

After approx 20 Years of $1/m_Q$ expansion

- ... we are able to perfome QCD-based appraoches
- ... many models have been superseeded
- ... and the model dependence has been pushed back into subleading terms
- ... we entered an era of precision flavour physics,
- ... at least for many quantities

Fruitful overlapp with other QCD based methods

- QCD Sum Rules
- Lattice QCD

Of course, there are problems ...

- Exclusive Non-Leptonic Decays
  Important for CP violation and searches for "New Physics"
- Tensions between exclusive and inclusive determinations of CKM parameters
- Proliferation of parameters in higher orders $1/m_b$
  Limits our ability to compute
- Higher orders in α_s are a real technical challenge
- More data may clarify a few of these things:
  LHCb and Superflavour ...