Lepton Flavour Violation $(\nu \to$ BSM EFT)

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1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
   - add light singlet ν_Rs to SM, Dirac mass partners of ν_L.
   - add non-renorm LNV operator $[\ell H][\ell H]$ to $\mathcal{L}_{SM}$
3. (m_ν observables and "mechanisms" ($\neq$ models))
4. not worry about origin of m_ν; assume leptonic NP with $\Lambda_{NP} \gtrsim m_W$, describable by $\mathcal{L}_{eff}$:

(only SM externallegs = neglect possibility of light ν_R)

$$\begin{aligned}
\mathcal{L}_{eff} &\simeq SM + maj.mass + 4ferm. + maj.mag.mo. + NS\nu I + \ldots \\
&\simeq \mathcal{L}_{SM} + \frac{K}{4M}(\ell H)(\ell H) + h.c. \\
&\quad -\frac{4G_F}{\sqrt{2}}\Big[\epsilon^{ijpr}_{\ell q(1)}(\overline{\ell}_i\gamma^\mu\ell_j)(\overline{q}_p\gamma^\mu q_r) + \ldots + \epsilon^{ijkn}_{\ell\ell(1)}(\overline{\ell}_i\gamma^\mu\ell_j)(\overline{\ell}_k\gamma_\mu\ell_n) + \ldots + \mu_{ij}\overline{\ell}_i H\sigma_{\mu\nu}e_{Rj}B^{\mu\nu} - \\
&\quad + \Big[\frac{C}{\Lambda^3}\ell_i H\sigma_{\mu\nu}\ell_j H B^{\mu\nu} + \ldots + h.c\Big] \\
&\quad + \Big[\epsilon G_F^2([\overline{\ell}H^*]\gamma^\mu[H\ell])(\overline{f}\gamma^\mu f) + \ldots + h.c\Big]
\end{aligned}$$

Outline (again)

1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
3. "mechanisms" ($\neq$ model) for small masses
4. **dim 6 in $\mathcal{L}_{eff}$: flavour changing interactions of the charged leptons (FCNC due to NP)**
   - where to look?
     - under the lamppost: where are the strong exptal/observational limits?
     - from the PDB to bounds on operator coefficients
   - pheno expectations? but there is no MFV??
   - bounds on your favourite model
     - tree level: $(\bar{Q}\Gamma Q)(\bar{L}\Gamma L) \leftrightarrow$ leptoquarks/RPV SUSY
     - loops: only charged leptons: your favourite neutrino mass mechanism
5. dim 7 and 8 neutrino operators

"Under the lamppost" (= in the PDB): what are good bounds on dim 6?

$$BR(K_L \to \mu\bar{e}) < 4.7 \times 10^{-12} \quad , \quad BR(B_d \to \mu\bar{\mu}) \lesssim 10^{-8}$$

$$\frac{BR(K^+ \to \pi^+\nu\bar{\nu}, \pi^+ l^+ l^-)}{BR(K^+ \to \nu\bar{\mu}\pi^0)} < \frac{1.7 \times 10^{-10}, 5 \times 10^{-10}}{5.1 \times 10^{-2}} \qquad l = e, \mu$$

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$$\frac{\Gamma(\mu \mathrm{Au} \to e \mathrm{Au})}{\Gamma(\mu \mathrm{Au} \to N' \nu)} < 7 \times 10^{-13} \qquad \frac{\Gamma(\mu \mathrm{Ti} \to e \mathrm{Ti})}{\Gamma(\mu \mathrm{Ti} \to N' \nu)} < 4 \times 10^{-12} \qquad Z_{Au} = 79, Z_{Ti} = 22$$

$$BR(\tau \to 3\ell) < 2 - 4 \times 10^{-8}$$

$$BR(\tau \to \mu \gamma) < 4 \times 10^{-8} \quad , \quad BR(\mu \to e \gamma) < 1.2 \times 10^{-11}$$

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But: small BR ⇔ LFV NP suppressed wrt SM (so appears less good if SM suppressed by m_i^2, θ^2) maybe want “absolute” bounds on the operator coefficients? ...a little work to do... Easier than quarks! At most 2 q legs at dim 6.

bounds on operator coefficients: $\tau \to \ell\gamma$

Straightforward to extract bounds on operator coefficients from radiative decays, because are mediated by (only a few) dipole operators:

$$\mathcal{O}_{eB}^{ij} = \overline{\ell}_i \sigma^{\mu\nu} e_{Rj} H B_{\mu\nu}, \qquad \mathcal{O}_{eW}^{ij} = \overline{\ell}_i \sigma^{\mu\nu} \tau^I e_{Rj} H W_{\mu\nu}^I.$$

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After SSB, get

$$\frac{C^{ij}}{\Lambda_{NP}^2} \langle H \rangle \overline{e_i} \sigma^{\alpha\beta} P_R e_j F_{\alpha\beta} + h.c. \quad = \quad \frac{e m_j}{2} \Big(\overline{e_i} \sigma^{\alpha\beta} (A_R^{ij} P_R + A_L^{ij} P_L) e_j F_{\alpha\beta} \Big)$$

Operator is chirality flip: $\propto$ (Yukawa)$^{2n+1}$. So explicit a power of m_j. ($[A] = 1/m^2$) .

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Can calculate :

$$\frac{\Gamma(\tau \to \mu\gamma)}{\Gamma(\tau \to \mu\nu\bar{\nu})} = \frac{e^2 m_\tau^5 (|A_L|^2 + |A_R|^2)}{16\pi} \frac{192\pi^3}{G_F^2 m_\tau^5} = \frac{48\pi^3\alpha}{G_F^2} \left(A_L^2 + A_R^2 \right) < \frac{4.4 \times 10^{-8}}{.17}$$

!! strong bound $(A_L^2 + A_R^2)/G_F^2 \lesssim 10^{-9}$!!

1. Not pay Yukawa for chirality flip: the dominant weak decay is via a dim 6 op, and m_j is the energy scale of the decay, so $\Gamma \propto G_F^2 m_j^5$.

2. Radiative decay pays a factor e^2, but enhanced wrt usual weak decay by (2 body phase space)/(3ody phase space) $\sim 2\pi^3$.

$2Q2L$ operators

A list of possible operators (from Flav@LHC Ybook)

$$\mathcal{O}^{ijkl}_{(1)\ell q} = (\overline{\ell}_i \gamma^\mu \ell_j)(\overline{q}_k \gamma_\mu q_l), \qquad \mathcal{O}^{ijkl}_{(3)\ell q} = (\overline{\ell}_i \tau^I \gamma^\mu \ell_j)(\overline{q}_k \tau^I \gamma_\mu q_l),$$

$$\mathcal{O}^{ijkl}_{ed} = (\overline{e}_i \gamma^\mu P_R e_j)(\overline{d}_k \gamma_\mu P_R d_l), \qquad \mathcal{O}^{ijkl}_{eu} = (\overline{e}_i \gamma^\mu P_R e_j)(\overline{u}_k \gamma_\mu P_R u_l),$$

$$\mathcal{O}^{ijkl}_{\ell u} = (\overline{\ell}_i u_l)(\overline{u}_k \ell_j) = -\frac{1}{2}(\overline{\ell}_i \gamma^\mu \ell_j)(\overline{u}_k \gamma_\mu P_R u_l)$$

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$$\mathcal{O}^{ijkl}_{\ell q S} = (\overline{\ell}_i e_j)(\overline{q}_k u_l) \qquad \mathcal{O}^{ijkl}_{qde} = (\overline{\ell}_i e_j)(\overline{d}_k q_l)$$

(operator normalisation à la Flavour@LHC Workshop Report de Mangano et al; Eur.Phys.J.C57:13-182,2008. *BUT N.B.*, typos in arXiv:0801.1826)

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Put in $\mathcal{L}$ as (aim to get F-rule $4G_F/\sqrt{2} \leftrightarrow C_X$)

$$\begin{aligned}
\mathcal{L} &= \ldots - \sum_{i,j,k,l=1}^{3} \left\{ C^{ijkl}_{\ell q S} \mathcal{O}^{ijkl}_{\ell q S} + C^{ijkl}_{qde} \mathcal{O}^{ijkl}_{qde} + h.c. \right\} \\
&\quad -\frac{1}{2} \sum_{i,j,k,l=1}^{3} \left\{ C^{ijkl}_{(1)\ell q} \mathcal{O}^{ijkl}_{(1)\ell q} + C^{ijkl}_{(3)\ell q} \mathcal{O}^{ijkl}_{(3)\ell q} + C^{ijkl}_{eu} \mathcal{O}^{ijkl}_{eu} + C^{ijkl}_{ed} \mathcal{O}^{ijkl}_{ed} + h.c. \right\} \\
&\quad -\frac{1}{2} \sum_{i,j,k,l=1}^{3} \left\{ -2C^{ijkl}_{\ell u} \mathcal{O}^{ijkl}_{\ell u} - 2C^{ijkl}_{\ell d} \mathcal{O}^{ijkl}_{\ell d} + h.c. \right\}
\end{aligned}$$

1/2 to compensate $+h.c.$ for hermitian ops. -ve sign to ressemble Fermi

Bounds on $2Q, 2L$ operator coefficients from leptonic meson decays

In the presence of SM gauge invariant operators(flavours i, j, k, n summed)

$$-\epsilon^{ijkn}_{(1)\ell q}\frac{4G_F}{\sqrt{2}}(\bar{\ell}^i\gamma^\mu P_L\ell^j)(\bar{q}^k\gamma_\mu P_L q^n) - \left\{\epsilon^{ijkn}_{qde}\frac{4G_F}{\sqrt{2}}(\bar{e}^i P_L\ell^j)(\bar{q}^k P_R d^n) + h.c.\right\}$$

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The decay rate of a pseudoscalar meson $M(\bar{q}_k d_n)$ is

$$\Gamma\left(M_{kn}\to l^i\bar{l}^j\right) = \frac{kG_F^2}{\pi m_M^2}\Bigg\{ \left[\epsilon^{ijkn}_{(1)\ell q}\right]^2\widetilde{A}^2\left[(m_M^2-m_i^2-m_j^2)(m_i^2+m_j^2)+4m_i^2m_j^2\right] + (\epsilon^{ijkn}_{qde})^2\widetilde{P}^2\left(m_M^2-m_i^2-m_j^2\right)+2\left[\epsilon^{ijkn}_{qde}\epsilon^{ijkn}_{(1)\ell q}\right]\widetilde{A}\widetilde{P}m_j\left(m_M^2+m_i^2-m_j^2\right)\Bigg\}$$

where its axial vector and pseudoscalar current matrix elements that contribute:

$$\widetilde{A}P^\mu = \frac{1}{2}\langle 0|\overline{q}\gamma^\mu\gamma^5 q|M\rangle = \frac{f_M P^\mu}{2} \qquad \widetilde{P} = \frac{1}{2}\langle 0|\overline{q}\gamma^5 q|M\rangle = \frac{f_M m_M}{2}\frac{m_M}{m_k+m_n}\ .$$

P^μ is the momentum of the meson, and k is the magnitude of the lepton 3-momentum in the center-of-mass frame:

$$k^2 = \frac{1}{4m_M^2}\left[\left(m_M^2-(m_i+m_j)^2\right)\left(m_M^2-(m_i-m_j)^2\right)\right]$$

Bounds from $K_L \to \mu\bar{e}$

A list of possible operators (maybe complete; see Flav@LHC Ybook) $(X = L, R\ )$

$$\epsilon_{Lq}^{\mu esd}\frac{2G_F}{\sqrt{2}}(\bar{\mu}\gamma^\mu P_X e)(\bar{s}\gamma_\mu P_L d) + \epsilon_{Ld}^{\mu esd}\frac{2G_F}{\sqrt{2}}(\bar{\mu}\gamma^\mu P_X e)(\bar{s}\gamma_\mu P_R d)$$

$$\epsilon_{Lq}^{\mu eds}\frac{2G_F}{\sqrt{2}}(\bar{\mu}\gamma^\mu P_X e)(\bar{d}\gamma_\mu P_L s) + \epsilon_{Ld}^{\mu eds}\frac{2G_F}{\sqrt{2}}(\bar{\mu}\gamma^\mu P_X e)(\bar{d}\gamma_\mu P_R s)$$

$$\epsilon_{qde}^{\mu esd}\frac{4G_F}{\sqrt{2}}(\bar{\mu}P_L e)(\bar{s}P_R d) + \epsilon_{qde}^{\mu eds}\frac{4G_F}{\sqrt{2}}(\bar{\mu}P_L e)(\bar{d}P_R s)$$

Obtain that

$$\begin{aligned} |\epsilon_{(1)\ell q}^{\mu esd}|^2 \quad &\to \quad \frac{1}{2}\Big|\epsilon_{\ell d}^{\mu esd} + \epsilon_{\ell d}^{\mu eds} - \epsilon_{\ell q}^{\mu esd} - \epsilon_{\ell q}^{\mu eds}\Big|^2 \\ &\quad\ + \frac{1}{2}\Big|\epsilon_{ed}^{\mu esd} + \epsilon_{ed}^{\mu eds} - \epsilon_{eq}^{\mu esd} - \epsilon_{eq}^{\mu eds}\Big|^2 \\ \epsilon_{qde}^{\mu esd} \quad &\to \quad \frac{1}{\sqrt{2}}\Big(\epsilon_{e\ell d}^{\mu esd} + \epsilon_{e\ell d}^{\mu eds}\Big) \end{aligned}$$

1/2 because $K_L = (|ds\rangle \pm |sd\rangle)/\sqrt{2}$

From the exptal bound on the Branching Ratio, get a bound:

1. BR sets bound on linear combo of coefficient of $\langle 0|\overline{q}\gamma^\mu\gamma^5 q|M\rangle$ and $\langle 0|\overline{q}\gamma^5 q|M\rangle$
2. but each coefficient is linear combo of coefficients of different SM gauge invar operators, for fermions of various chiralities.
3. and anyway, your NP maybe didn't give those nice current current $V \pm A$ operators, maybe you had to do Fiertz to get that form,so operator coefficients are linear combos of NP coefficients.

repeat for all LFV rates in PDB... WELCOME TO THE ZOO !!

Brute force extraction of bounds on NP operator coefficients from data is a mess....

$\Rightarrow$ set bounds on operator coefficients by turning on one operator at a time
(but remember this misses possible cancellations $\leftrightarrow$ depends on the choice of operator basis.)

Expectations? MFV(?L?)

- More clever approach: identify operators "expected" to dominate, and set bounds on their coefficients.
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- ⇒ no bottom-up pheno defn of MFVL.
  (Several models in the literature called MFVL).

Summary

extracting bounds on operator coefficients from data is a can of worms = not very enlightening (even though more doable than quark sector)

A la différence des quarks, no SM LFV
$\Rightarrow$ no MFV-like expectations for an SM pattern of LFV

beautiful machinery of EFT not required : QED running :GeV $\longrightarrow m_W$ is small, not need EW running : $m_W \longrightarrow \Lambda_{NP}$ if want $\Lambda_{NP} \sim$ TeV.

A la différence des quarks, $know$ there is NP

Dans le meilleur des mondes possibles: bottom-up reconstruction of NP from the coefficients $\{C_X\}$

In practise: ...models. (skip EFT, just compute rates in your favourite model).

Models: a leptoquark

Consider, for instance, a singlet scalar "leptoquark" $\tilde{\bar{\tilde{D}}}$, with interactions:

leptoquark $\mathcal{L}$ in
Buchmuller, Wyler NPB 1986

$$\tilde{\bar{\tilde{D}}}\lambda\overline{q_L^c}\epsilon\ell = [\lambda]^{lq}\tilde{\bar{\tilde{D}}}(\overline{u_L^c}_q e_l - \overline{d_L^c}_q \nu_l)$$

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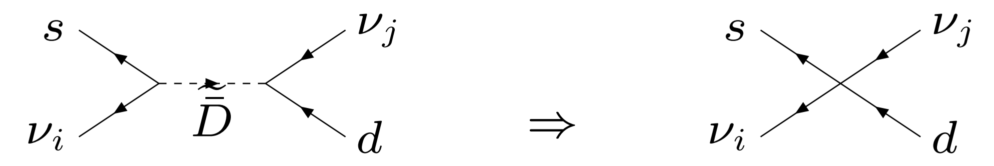
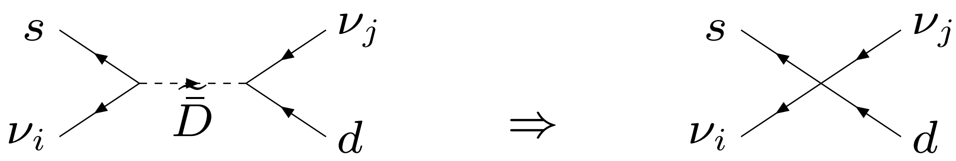
Obtain such interactions, if include the lepton number violating λ' coupling in an R-parity violating addition to the MSSM superpotential:

$$\mathcal{W}_{RPV} = \lambda_{lq}^{\prime k} L_l Q_q \bar{D}_k \rightarrow \lambda_{lq}^{\prime k}\tilde{\bar{D}}_k(\overline{u_{L_q}^c}e_l - \overline{d_{L_q}^c}\nu_l) + \ldots$$

If all doublet squarks and two generations of singlets are negligeably heavy, then only include $\tilde{\bar{D}}$ exchange which gives effective four-fermion vertex:



Models: a leptoquark, and $K^+ \to \pi^+ \nu\bar{\nu}$



which can be re-expressed as a $(V - A)$ product of quark and lepton currents:

$$\frac{\lambda'^*_{is}\lambda'_{jd}}{m^2_{\tilde{D}}}(\overline{s^c}P_L\nu_i)(\overline{\nu}_jP_Rd^c) = -\frac{\lambda'^*_{is}\lambda'_{jd}}{2m^2_{\tilde{D}}}(\overline{s}\gamma^\rho P_Ld)(\overline{\nu}_i\gamma_\rho P_L\nu_j) = \frac{4G_F}{\sqrt{2}}\varepsilon^{jisd}(\overline{s}\gamma^\rho P_Ld)(\overline{\nu}_i\gamma_\rho P_L\nu_j)$$

which can contribute to $K^+ \longrightarrow \pi^+\nu\bar{\nu}$.
ν flavour undetected. NP with $i = j$ interferes with the SM, for $i \neq j$, | NP amplitude$|^2$ is bounded.
(measurement is $\sim 2\times$ the SM expectation, $\Rightarrow$ bounds of same order):

$$|\varepsilon^{jisd}| \lesssim 10^{-5}$$

(recall $\lambda'_{ib} \sim 10^{-3} - 10^{-4}$ to generate $[m_\nu]$.)

Some Signs and Fiertz Transformations

Relative sign between scalar/vector mediated 4-f ops:

$$\frac{i}{p^2 - m^2} \to \frac{-i}{{m_L}^2} \quad \text{and} \quad \frac{-ig^{\mu\nu}}{p^2 - m^2} \to \frac{+ig^{\mu\nu}}{{m_L}^2}$$

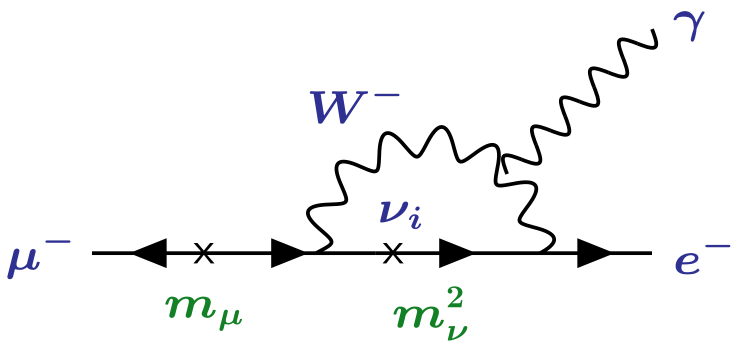
Useful identities for transforming 4-fermion operators into a form $\sim (V - A)(V - A)$ of weak interactions (allows to normalise NP rates to SM rates). Simplest in 2comp notn!

$$\begin{aligned}
(\overline{a^c} P_L b) &= (\overline{b^c} P_L a) & [\chi\psi = \psi\chi] \\
(\bar{a} P_L b)^\dagger &= (\bar{b} P_R a) & [(\psi\chi)^\dagger = \bar{\psi}\bar{\chi}] \\
(\bar{a}\gamma^\mu P b)^\dagger &= (\bar{b}\gamma^\mu P a) & [(\chi\sigma\bar{\psi})^\dagger = \psi\sigma\bar{\chi}] \\
(\overline{a^c}\gamma^\mu P_{L,R} b^c) &= -(\bar{b}\gamma^\mu P_{R,L} a) & [\chi\sigma\bar{\psi} = -\bar{\psi}\bar{\sigma}\chi]
\end{aligned}$$

And Fiertz ($\leftrightarrow$ SU(N) identity: $T^A_{\alpha\beta} T^A_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$, for SU(2) ($T = \sigma/2$) $\sigma^i_{\alpha\beta}\sigma^i_{\gamma\delta} + \delta_{\alpha\beta}\delta_{\gamma\delta} = +2\delta_{\alpha\delta}\delta_{\gamma\beta}$) :

$$\begin{aligned}
(\bar{a} P_L b)(\bar{c} P_R d) &= -\frac{1}{2}(\bar{a}\gamma^\mu P_R d)(\bar{c}\gamma_\mu P_L b) \\
(\bar{a}\gamma^\mu P_{L,R} b)(\bar{c}\gamma_\mu P_{L,R} d) &= (\bar{a}\gamma^\mu P_{L,R} d)(\bar{c}\gamma_\mu P_{L,R} b)
\end{aligned}$$

$\mu \to e\gamma$ in presence of m_ν



Gives multiplicative GIM suppression (no log... :(...ν not couple to γ ...)

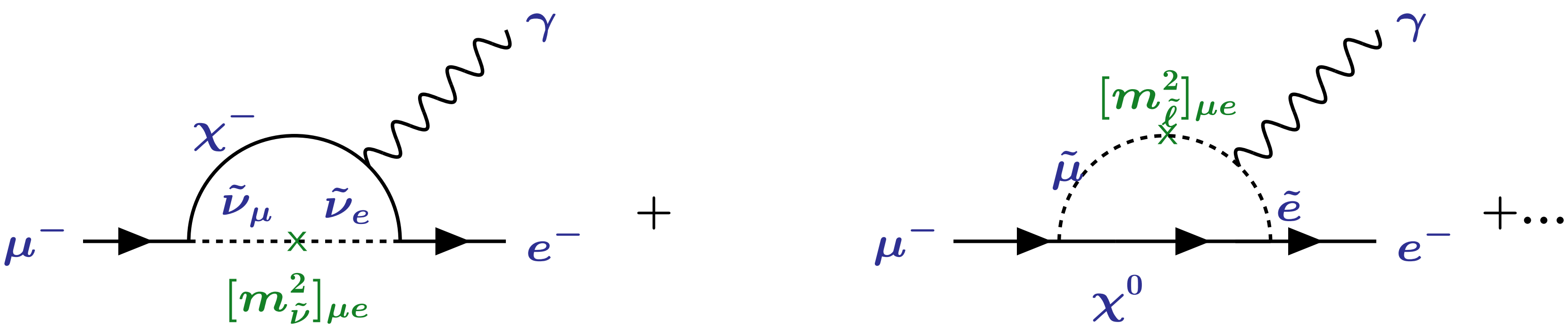
$$m_\mu A_L \sim m_\mu G_F \frac{e}{16\pi^2} \frac{U_{\mu i} m_{\nu,i}^2 U_{ei}^*}{m_W^2}$$

$$\Rightarrow BR(\mu \to e\gamma) \simeq \left(\frac{m_\nu}{m_W}\right)^4$$

Exercise: compute (unitary gauge), A_L in the SM with massive neutrinos

A detectable $\mu \to e\gamma$ rate: the SUSY See-Saw

sparticle loops :



$$m_\mu A_L \sim \frac{m_\mu}{m^2_{SUSY}} \frac{g^2 e}{16\pi} \frac{[m^2_{\tilde{\nu}}]_{\mu e}}{m^2_{SUSY}} + \ldots$$

SUSY param dep
Graesser Thomas 2001

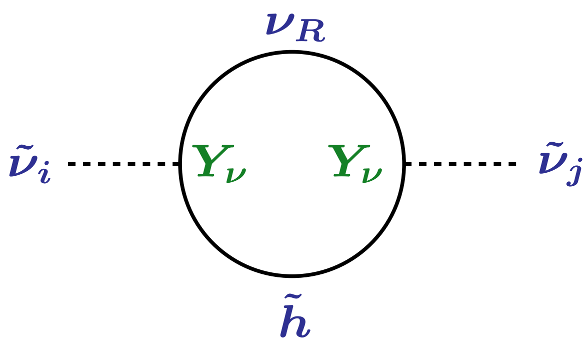
suppressed by LFV soft masses, rather than m_ν:

$$BR(\mu \to e\gamma) < 10^{-11} \quad \Rightarrow \quad \frac{[m^2_{\tilde{\nu}}]_{\mu e}}{m^2_{SUSY}} \lesssim 2 \times 10^{-3}$$

(in the mass insertion approximation)

LFV slepton masses from RGE — not suppressed by M^{-1}

- suppose soft scalar masses universal at M_{GUT}: $\sim m_o^2 \mathbf{I}$
- Renormalisation Group running $M_{GUT} \to M_{3,2,1}$ will induce flavour violation at the weak scale in slepton masses:

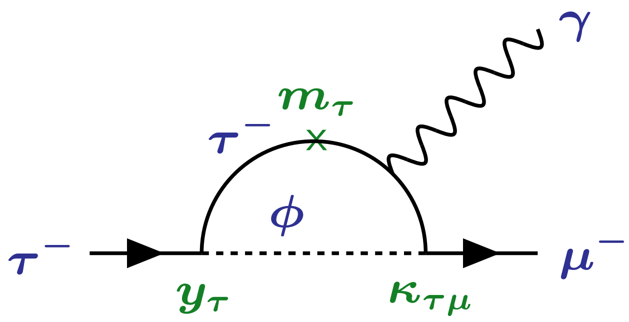


$$\left[m^2_{\tilde{\nu}}\right]_{ij} \quad \simeq \quad \text{(diag part)} - \frac{3m_0^2 + A_0^2}{8\pi^2} (\mathbf{Y}_\nu^\dagger)_{ik} (\mathbf{Y}_\nu)_{kj} \log \frac{M_{GUT}}{M_k}$$

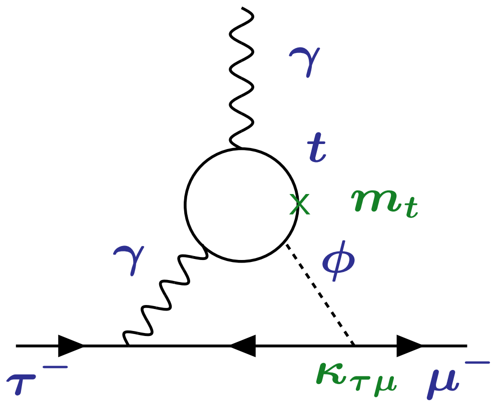
log GIM! (well, in the ν_R Majorana masses)

Detectable $\tau \to \mu\gamma$: put LFV "by hand"

Consider "type III" 2HDM = allow flavour changing interaction $\sim \kappa_{\tau\mu}\phi\bar{\tau}\mu$ for neutral Higgses $\phi = h, H, A$. (Rappelle: (the MSSM is type II at tree: d and e couple to H_d, u couple to H_u.
but at one loop, the MSSM is type III — loop diagrams connect fermions to the (wrong Higgs)†)



$$m_\tau A_X \sim \frac{e\kappa_{\tau\mu}y_\tau}{16\pi^2}\frac{m_\tau}{m_\phi^2}$$



$$m_\tau A_X \sim \frac{e^3\kappa_{\tau\mu}y_t}{(16\pi^2)^2}\frac{m_t}{m_\phi^2}$$

$$\frac{m_\tau A_X|_{1\ \mathrm{loop}}}{m_\tau A_X|_{2\mathrm{loop}}} \sim \frac{16\pi^2 y_\tau m_\tau}{N_c y_t m_t} < 1 \qquad (\mathrm{modulo}\frac{1}{\tan\beta}\mathrm{factors\ in\ } y_t)$$

moral: 2 loop can be relevant when enters top and QCD

Trying to keep up with Uli: hierarchical Zs and $\mu \to e\gamma$

Recall that, in principle,

1. write leptonic Lagrangian:

$$i\overline{\ell}_a[Z_\ell Z_\ell^\dagger]^{ab} \not{D}\ \ell_b + i\overline{e}_a[Z_e Z_e^\dagger]^{ab} \not{D}\ e_b + i\overline{e}_a[\tilde{Y}]^{ba}](\ell_b H) + h.c.$$

2. diagonalise $ZZ^\dagger$
3. rotate/renormalise SM fields to get canonical kinetic terms $i\overline{\psi}^a \not{D}\ \psi_a$
4. diagonalise $Y_e = Z_\ell^{-1}\tilde{Y}]Z_e^{-1}$ by unitary transformations in ℓ and e flavour spaces.

In the renormalisable SM, can always redefine fields to obtain in herarchy in the Yukawas or the Zs. Observable is the "relative" hierarchy $\sim Z^{-1}\tilde{Y}Z^{-1}$.

Trying to keep up with Uli:hierarchical Zs and $\mu \to e\gamma$

Recall that, in principle,

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Usually, you did this before you added the higher dim operators that parametrise NP. And you had a "flavour problem". Which you solvee in quark sector with MFV.

Trying to keep up with Uli: hierarchical Zs and $\mu \to e\gamma$

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Usually, you did this before you added the higher dim operators that parametrise NP. And you had a "flavour problem". Which you solved in quark sector with MFV.

Suppose instead:

1. you put the NP operators before making kinetic terms canonical
2. you allow $\tilde{Y}$ and C_X to have $\mathcal{O}(1)$ coefficients, for any flavour combinations
3. you put the observed yukawa hierarchy in the eigenvalues of Z^{-1}:

$$\frac{1}{z_{e_L}} \simeq \sqrt{y_e}$$

and *then* you renormalise to obtain canonical kin terms, and diagonalise Yukawas.

$\Rightarrow$ the Yukawa hierarchy is imposed on all higher dimensional operators

What as this got to do with Uli?

This is the 4-d, EFT relative of wavefn overlaps in extra dims, that gives "natural" suppression of flavour violation.

So the recipe is:

1. write leptonic Lagrangian:

$$i\overline{\ell}_a[Z_\ell Z_\ell^\dagger]^{ab} \not{D} \ \ell_b + i\overline{e}_a[Z_e Z_e^\dagger]^{ab} \not{D} \ e_b + i\overline{e}_a[\tilde{Y}]^{ba}(\ell_b H) + h.c. + \sum \frac{C_X}{\Lambda^n}\mathcal{O}_X$$

2. Data tells you there is a relative hierachy between Zs and Ys. *Put it in* Z: allow $\tilde{Y}$ and C_X to have $\mathcal{O}(1)$ coefficients, and put the hierarchy in the eigenvalues of Z^{-1}:

$$\frac{1}{z_{e_R}} \simeq \frac{1}{z_{e_L}} \simeq \sqrt{y_e} \qquad \frac{1}{z_{\mu_R}} \simeq \frac{1}{z_{\mu_L}} \simeq \sqrt{y_\mu}$$

3. diagonalise $ZZ^\dagger$
4. rotate/renormalise SM fields to get canonical kinetic terms $i\overline{\psi}^a \not{D} \ \psi_a$
5. diagonalise $Y = Z_L^{-1}\tilde{Y}Z_R^{-1}$ by unitary transformations in ℓ and e flavour spaces.
6. Suppose that

$$\frac{C}{\Lambda^2} \lesssim \frac{g^2}{16\pi^2(3m_Z)^2} \sim \frac{1}{(10TeV)^2}$$

   and discover that your only (mild) quark flavour problem is ϵ_K.

Does it work

From ϵ_K, get bound on coeff $C_{LR2}/(10TeV)^2$ of $(\overline{d_R}s_L)(\overline{d_L}s_R)$:

$$|C^{21}_{LR2}| \sim \frac{1}{z_q^{(2)} z_q^{(1)} z_d^{(2)} z_d^{(1)}} < \ 0.004\ |V^*_{ts}V_{td}|^2 \approx 0.6 \times 10^{-9}$$

whereas expected:

$$\frac{1}{|z_q^{(2)} z_q^{(1)} z_d^{(2)} z_d^{(1)}|} \sim \frac{m_d m_s}{v^2} \approx 1 \times 10^{-8}\ .$$

... what is your defn of 1? ...need $[\mathcal{O}(1)$ factors$]^4 \lesssim 1/20$

(simple Froggart Nielson, with $1/z_A^{(i)} \sim \epsilon^{Q^i_A}$,

$$C^{ij}_{AB} \to \epsilon^{|Q^i_A - Q^j_B|}$$

Hierarchies give:

$$C^{ij}_{AB} \to \left(Z_A^{-1} C_{AB} Z_B^{-1}\right)^{ij} \ \sim \ \frac{1}{z_A^{(i)}\ z_B^{(j)}} \quad (A, B \in \{\text{SM fermions}\}, i, j\ flavour)$$

a bit more suppressed...

Back to $\mu \to e\gamma$

Expect large rates for $\Delta F = 1$ processes due to non-renorm operators that are bilinear in the lepton fields (suppressed only by two $z_{L,E}$ factors):

$$\mathcal{L} = \frac{C^{ij}_{RL1}}{\Lambda^2} g' H^\dagger \overline{e_R}^i \sigma^{\mu\nu} \ell^j_L B_{\mu\nu} + \frac{C^{ij}_{RL2}}{\Lambda^2} g H^\dagger \overline{e_R}^i \sigma^{\mu\nu} \tau^a \ell^j_L W^a_{\mu\nu} + h.c.$$

Then ($e y_\mu A_R \simeq C^{\mu e}_{RL\gamma}/\Lambda^2 = (C^{\mu e}_{RL2} - C^{\mu e}_{RL1})/\Lambda^2$ and $C^{\mu e} \sim \sqrt{y_\mu y_e}$)

$$\begin{aligned} BR(\mu \to e\gamma) &= \frac{192\,\pi^3\,\alpha}{G_F^2 \Lambda^4} \frac{1}{y_\mu^2} \left[|C^{\mu e}_{RL\gamma}|^2 + |C^{e\mu}_{RL\gamma}|^2 \right] \\ &\approx 1.2 \times 10^{-11} \left(\frac{130 \text{ TeV}}{\Lambda} \right)^4 \qquad !!!!! \end{aligned}$$

(Expectations for $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are with in exptal bounds for $\Lambda \sim 10$ TeV.)

??? the scale of the LFV operators is pushed well above $10\ \mathrm{TeV}$ or, additional suppression...

But extra dim models do better: if the dipole operator is generated only via an effective four-lepton interaction (with two lepton lines closed into a loop), its coupling receives an extra suppression factor $\sim y_\mu$ which allow to set $\Lambda \sim 10$ TeV.

Dimension 7: majorana neutrino mag mos, etc

Dim 5 magnetic moment interaction $[\mu] = 1/\text{mass})$:

$$\frac{\mu_{ij}}{2}\overline{\psi}_i \sigma^{\mu\nu}\psi_j F_{\mu\nu} \quad \rightarrow \quad \frac{\mu_{ij}}{2}\,\overline{\nu^c}_i \sigma^{\mu\nu} P_L \nu_j (F_{\mu\nu}) + h.c.$$

flips the chirality of the fermion passing through, vanishes for $i = j$: $[\mu]_{ij} = -[\mu]_{ji})$ ψ is a four-component fermion, $\overline{\nu^c} = (-i\gamma_2(\nu^\dagger)^T)^\dagger \gamma_0$,

Two possible dimension seven operators which give a neutrino magnetic moment interaction after SSB:

$$[O_B]_{\alpha\beta} = g'(\overline{\ell^c}_\alpha \epsilon H)\sigma^{\mu\nu}(H\epsilon P_L \ell_\beta)B_{\mu\nu}, \quad [O_W]_{\alpha\beta} = ig\varepsilon_{abd}(\overline{\ell^c}_\alpha \epsilon \tau^a \sigma^{\mu\nu}\ell_\beta)(H\epsilon\tau^b H)W^d_{\mu\nu}.$$

$\{\tau_i\}$ are the SU(2) Pauli matrices, the SU(2) contractions are implicit in the parentheses ($\epsilon = -i\tau_2$, $(v\epsilon u) = v_2 u_1 - v_1 u_2$), $\varepsilon_{abd} \neq \epsilon$ is the totally antisymmetric tensor, and $W_{\mu\nu}$, $B_{\mu\nu}$ are the gauge field strength tensors for SU(2) and U(1)$_Y$.

They are potentially interesting, because there is a mild anti-correlation of sunspot activity (solar $\vec{B}$ fields) and solar ν_e flux... which can explain with $\mu_\nu \lesssim$ current upper bd

Notice that they are lepton number violating, like majorana masses...

Pheno bounds on majorana neutrino mag mos

1. $(\Gamma(\nu_j \to \bar{\nu}_i \gamma) \propto m_\nu^N$
2. bounds from ν scattering experiments:

$$2\mu_{e\beta} \leq 0.9 \times 10^{-10} \mu_B, \quad 2\mu_{\mu\beta} \leq 6.8 \times 10^{-10} \mu_B, \quad 2\mu_{\tau\beta} \leq 3.9 \times 10^{-7} \mu_B \qquad \text{expt}$$

   (γ exchange can enhance over Z at p_T)
   2 is because our neutrinos are majorana
3. in a stellar plasma, "decay" of photons into ν pairs: $\gamma \to \nu_\alpha \nu_\beta$ allows E_γ to escapes the star. cooling rate of globular cluster stars:

$$2[\mu]_{\alpha\beta} \lesssim 3 \times 10^{-12} \mu_B \qquad \text{astro} \ .$$

Dimensional analysis with majorana neutrino mag mos

$m_\nu \sim .1eV$ is "small": $(H\ell)(H\ell)$ induces neutrino masses $m_\nu \sim .1eV$ then the New Physics scale where this operator is generated should be $\lesssim v^2/(.1eV) \sim 10^{14}$ GeV.

whereas $\mu \sim 10^{-12}\mu_B$ is "large"

$$\mu \sim C_J v^2 \sim \frac{g^2/(16\pi^2)}{M^3} v^2 \sim 10^{-12}\mu_B$$

$$M^3 \lesssim 5 \times 10^{11} GeV^3 \left(\frac{10^{-12}\mu_B}{\mu} \right)$$

$M \lesssim 10$ TeV, if it is the same mass scale cubed.
If $M^3 \sim m_W^2 M_{max}$ (but how to build this model?), $\Rightarrow M_{max} \lesssim 10^8$ GeV.
$\Rightarrow$ Naive Dim Analysis says μ_ν unobservable small
$\Rightarrow$ ask the question other way round: is such a large mag mo consistent with small masses?

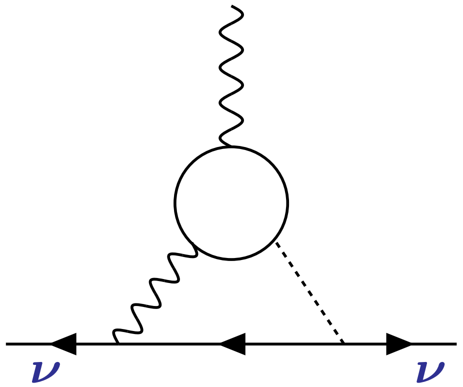
NB: μ_ν measured as frction of electron magnetic moment $\mu_B = e/(2m_e)$. For e, momentum in loops (contributing, $e.g.$ to $g-2$) is $1/p^2 \sim 1/m_e^2$, and m_e must appear upstairs to flip chirality. For ν, might expect $1/p^2 \sim 1/m_W^2$, suppressing $\mu_\nu \sim (m_e^2/m_W^2)\mu_B$. So $\mu_\nu \sim 10^{-12}\mu_B$ suggests lepton number violation near the weak scale.

Some models with measurable majorana mag mos

Models of measurable μ: if the photon is removed from the diagrams, it would naively seem that the dimension five neutrino mass operator is obtained, with a "natural" coefficient of order the inverse new physics scale. Need to suppress/forbid this dim 5 mass operator.

Voloshin: $[\mu]_{\alpha\beta}$ is flavour *antisymmetric*: arrange cancellations among the diagrams contributing to the flavour *symmetric* mass matrix.

Barr,Freire,Zee: forbids by angular momentum conservation the magnetic moment diagram with its photon removed. (Barr-Zee 2 loop diagram: vanishes if only 1 γ on fermion loop).



Georgi, Randall: recipe for forbidding diagrams: attribute a discrete quantum number, such that is conserved by mag mo, violated by mass. Introduce new physics respecting the sym that generates the mag mo. So then the new physics only contributes to the mass operator via higher order loops involving SM fields who don't respect the sym...

EFT: bounds on mag mo from RG mixing to dim 7 mass operator?

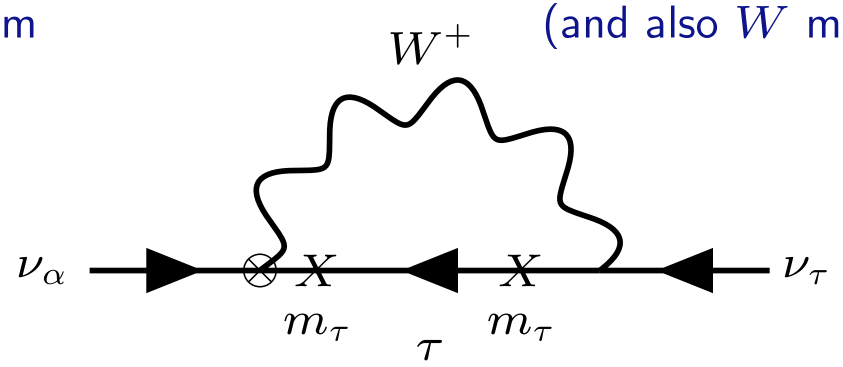
Suppose ν mag mo arises from $[O_W]_{\alpha\beta}$,

$$[O_W]_{[\alpha\beta]} = ig\varepsilon_{abd}(\overline{\ell^c}_\alpha \epsilon \tau^a \sigma^{\mu\nu} \ell_\beta)(H\epsilon\tau^b H)W^d_{\mu\nu} \rightarrow \text{Feynman rule for } \nu\tau{-}W^+ \sim \frac{Cv^2}{\Lambda^3}\sigma_{\mu\nu}k^\mu$$

(where $\mu_\nu \simeq Cv^2/\Lambda^3$). Does it mix to dim 7 mass operator (RG running $\Lambda \rightarrow m_W$)

$$[O_M]_{\{\alpha\beta\}} = (\overline{\ell^c}_\alpha \epsilon H)(H\epsilon\ell_\beta)(H^\dagger H)$$

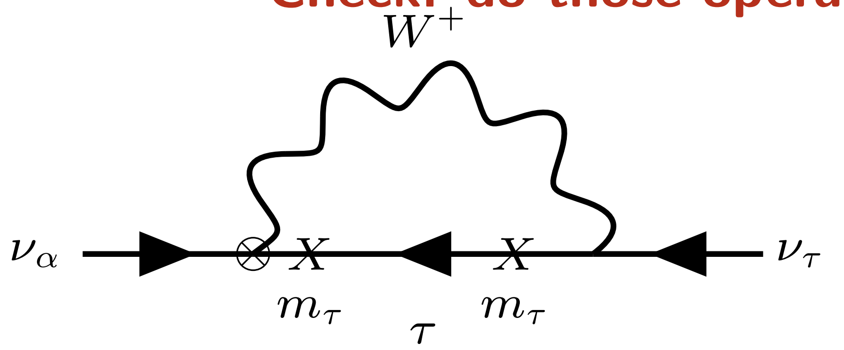
via diagram (and also W mag mo at other vertex):



NB: must have Yukawa insertions, because $[\mu]_{\alpha\beta}$ is flavour anti-sym, whereas $[m]_{\alpha\beta}$ is sym, so at best can get

$$\delta[m]_{\alpha\beta} \sim \frac{g}{16\pi^2}\mu_{\alpha\tau}|m_\alpha^{e2} - m_\tau^{e2}| \log\frac{\Lambda_{NP}^2}{m_W^2}$$

Check: do those operators really mix?



Guestimate (zero external momentum, no 2s, $m_W \to 0$ since):

$$\sim \quad g^2 \frac{C_W v^2}{\Lambda^3} \int \frac{d^4k}{(2\pi)^4} \sigma_{\mu\nu} k^\mu \frac{\not k}{k^2} \frac{m_\tau^2}{k^2} \gamma^\nu \frac{1}{k^2}$$

$$\sim \quad \frac{ig^2}{2}[\mu]_{\alpha\tau} \int \frac{d^4k}{(2\pi)^4} [\not k \ \gamma^\nu - \gamma^\nu \not k \ ] \frac{\not k}{k^2} \frac{m_\tau^2}{k^2} \gamma^\nu \frac{1}{k^2} \propto g^2 [\mu]_{\alpha\tau} m_\tau^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4}$$

a log div! Add diagram with mag mo at ν_τ vertex, gives

$$\delta[m]_{\alpha\beta} \sim \frac{g^2}{16\pi^2} \mu_{\alpha\tau} |m_{e\alpha}^2 - m_\tau^2| \log \frac{\Lambda_{NP}^2}{m_W^2}$$

Is the bound interesting?
Marginally: if hierarchical m_ν, a $\mu_{e\tau}$ relevant to solar physics (= can fit variation of solar ν flux with solar cycle), overcontributes to $[m_\nu]_{e\tau}$ by factor ~ 10.

Pandora's Box of Fermion Horrors : $[\mu]_{ij}$ is flav antisym

Use Peskin conventions, and notation. (NB: W+B use metric $(-,+++)$.) Take

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (1)$$

and

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu], \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (2)$$

where $\bar{\sigma}^\mu = (\sigma^0, -\sigma^i)$.

So $\gamma^{0\dagger} = \gamma^0$, and $\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu$.

Fermions anti-commute, cad they are grassman, but NB complex conjugation of grassman numbers is defined such that $(\alpha\beta)^* = \beta^*\alpha^*$.

A basis for 4×4 Dirac matrices is $\{I, \gamma^\mu, \gamma^\mu\gamma^5, \gamma^5, \sigma^{\mu\nu}\}$; according to Haber and Kane Appendix D, these have property that $\Gamma = \gamma^0\Gamma^\dagger\gamma^0$.

To convert from 4-component notn to 2...:

A 4-comp fermion ψ_D can be written as two chiral 2-comp fermions (LH $= \chi$, and RH $= \bar{\eta}$):

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\eta}^{\bar{\beta}} \end{pmatrix}$$

The 2-comp indices α and $\bar{\beta}$ run from 1..2, and are contracted with the anti-sym epsilon tensor

$$\varepsilon_{\bar{\alpha}\bar{\beta}} = \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon^{\bar{\alpha}\bar{\beta}} = \varepsilon_{\alpha\beta} = -\varepsilon^{\alpha\beta}$$

NB sign flip in going from dotted to undotted (barred in my incompetent latex) indices.

Undotted indices are always contracted up-down:

$$\chi\rho = \chi^\alpha \rho_\alpha = \varepsilon^{\alpha\beta}\chi_\beta\rho_\alpha = -\rho_\alpha\chi^\alpha = \rho^\alpha\chi_\alpha$$

and dotted indices down-up, and the ε flips sign in getting bars (sign flip because of up-down vs down-up summing conventions: $\bar{\rho}^{\bar{\beta}} = \bar{\rho}_{\bar{\alpha}}\varepsilon^{\bar{\alpha}\bar{\beta}}$, but $\bar{\rho}^{\bar{\beta}} = (\rho^\beta)^* = (\rho_\alpha\varepsilon^{\beta\alpha})^*$):

$$\begin{aligned} (\eta\rho)^* &= (\eta\rho)^\dagger = (\varepsilon^{\alpha\beta}\eta_\alpha\rho_\beta)^* = (-\varepsilon^{\bar{\alpha}\bar{\beta}})\bar{\rho}_{\bar{\beta}}\bar{\eta}_{\bar{\alpha}} \\ &= \bar{\rho}_{\bar{\alpha}}\bar{\eta}^{\bar{\alpha}} \end{aligned}$$

So, eg

$$\bar{\psi}_D = \begin{pmatrix} \bar{\chi}_{\bar{\alpha}} & \eta^\beta \end{pmatrix} \begin{pmatrix} 0 & \delta^{\bar{\alpha}}_{\bar{\rho}} \\ \delta^\omega_\beta & 0 \end{pmatrix} = \begin{pmatrix} \eta^\omega & \bar{\chi}_{\bar{\rho}} \end{pmatrix}$$

In practise, there is a -ve sign from interchanging fermion fields in an operator, but not when you take cc of the op.

The mag mo op...

For a generic Dirac fermion (coeff a_{ij} need not be antisym—fortunately, muon has mag mo)

$$
\begin{aligned}
a_{ij}\bar{\psi}_i\sigma^{\mu\nu}\psi_j F_{\mu\nu} \quad +h.c. = & \quad a_{ij}\bar{\psi}_i\sigma^{\mu\nu}\psi_j F_{\mu\nu} + a^*_{ij}\bar{\psi}_j\sigma^{\mu\nu}\psi_i F_{\mu\nu} \\
= & \quad \frac{i}{2}a_{ij}(\psi_{Ri})^\dagger\gamma^0[\gamma^\mu,\gamma^\nu]\psi_{Lj}(2q_\mu A_\nu) + \frac{i}{2}a_{ij}(\psi_{Li})^\dagger\gamma^0[\gamma^\mu,\gamma^\nu]\psi_{Rj}(2q_\mu A_\nu) \\
+ & \quad \frac{i}{2}a^*_{ij}(\psi_{Rj})^\dagger\gamma^0[\gamma^\mu,\gamma^\nu]\psi_{Li}(2q_\mu A_\nu) + \frac{i}{2}a^*_{ij}(\psi_{Lj})^\dagger\gamma^0[\gamma^\mu,\gamma^\nu]\psi_{Ri}(2q_\mu A_\nu)
\end{aligned}
$$

Now if impose that fermion is majorana, then in 4-comp notn this means $\psi^c_M = \psi_M$, where

$$
\psi^c = C\psi C = -i\gamma^2\psi^* = -i(\bar{\psi}\gamma^0\gamma^2)^T \quad , \qquad C = -C^T, \;\; C^{-1} = C^\dagger, \;\; C^\dagger\Gamma C = \pm 1\Gamma^T
$$

$\{\Gamma\}$ are the 16 basis matrices, and -ve sign under C is for the $\sigma^{\mu\nu}$. In 2-comp notn:

$$
\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\bar{\beta}} \end{pmatrix}
$$

With all this mess, and using commutation relns of σ matrices: $[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k$ it is easy to check that the mag mo coupling of same flavour majorana fermions vanishes. Roughly, this follows because $\psi^\dagger_R \sim \psi_L$, so the 2nd and 4th terms in mag mo interaction are the h.c. of the f1st and 3rd, and 1st+2nd is the same as 3rd+4th , but with the fermion order interchanged...and that interchange produces a minus sign...

Dimension 8: Non Standard (ν) Interactions (NSI)

At high intensity future ν facilities (νFact?), could have a beam of pure ν_μ (produce, collimate and cool (anti)muons, then store in a racetrack where they decay).

Measure all possible oscillation probablilites $\mathcal{P}_{\alpha\mu}(L) = |\mathcal{A}_{\alpha\mu}(L)|^2$ (at different distances L)

$$\mathcal{A}_{\alpha\mu}(L) = U_{\alpha 1}U^*_{\mu 1} + U_{\alpha 2}U^*_{\mu 2}e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{\alpha 3}U^*_{\mu 3}e^{-i(m_3^2 - m_1^2)L/(2E)}$$

Beam travels underground, "matter effect" must be included in neutrino "mass matrix" : ν_e are slowed down relative to $\nu_{\mu,\tau}$ because ν_e have CC and NC interactions with e of matter.

$\Rightarrow$ sensitivity to $\sin\theta_{13}$, δ, sign of Δm^2_{23}... as long as neutrinos don't have other "non-standard" interactions?

NSI in the in the CC interactions of production and detection — put a "near" detector, baseline to short for oscillations, and look for wrong flavour charged leptons.

Non Standard (ν) Interactions (NSI) that give NS matter effect?

Question: can you put a *new* interaction

$$\mathcal{L}_{eff}^{NSI} = -\varepsilon_{ij}^{fP} 2\sqrt{2} G_F (\bar{\nu}_i \gamma_\rho L \nu_j)(\bar{f} \gamma^\rho P f) \qquad (f = u, d, e)$$

with coeff $\varepsilon \gtrsim 10^{-3}$?

At dim 6, such operators are accompagnied by CC or charged lepton interactions (current expts have better sensitivity).

But, at dim 8!, for instance:

$$\bar{e}_R (H^\dagger \sigma^a \ell)(\bar{\ell} \sigma^a H) e_R \rightarrow -\frac{1}{2} \langle H \rangle^2 (\bar{e} \gamma^\rho R e)(\bar{\nu} \gamma_\rho L \nu)$$

with

$$\varepsilon_{ij}^{fP} 2\sqrt{2} G_F = \frac{C v^2}{\Lambda^4} \Leftrightarrow \varepsilon \sim \frac{v^4}{\Lambda^4}$$

So need NP at $\sim$ TeV (?see at LHC?), that should not generate dim 6 (Exercise: 1) build such a model. 2) publish.

From EFT perspective: can one show that pre-νFact expts will be sensitive to $\varepsilon \lesssim 10^{-3}$? Do these operators mix to dim 8 charged lepton operators?

...can show: finite terms but no log. :(