Effective Field Theories for Heavy Quarks

Part II: Heavy Quark Effective Theory

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Literature for Part II

- M. Luke, TASI Lectures TASI-02, 2004
- M. Wise, hep-ph/9805468
- M. Neubert, Phys. Rept 1993
- A Manohar, M. Wise,
  Heavy Quark Physics, Cambridge UP
- T. Mannel, Springer Tracts 2005

Contents Part II

What is the Problem?

- Weak interaction: Transitions between quarks
- Observations: Transitions between Hadrons
- Deal with QCD at large distance / small scales
- Extraction of fundamental parameters (CKM Elements, CP Phases) requires precise predictions, including error estimates $\to$ Simple models are out ...
- Heavy Quark Masses m_b and m_c are still perturbative scales
- $\to$ Matrix elements have perturbative contributions
- Extract these using effective field theory methods

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b,\ m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \qquad Q = b, c$$

- In this limit we have

$$\left.\begin{array}{l} m_{Hadron} = m_Q \\ p_{Hadron} = p_Q \end{array}\right\} v_{Hadron} = v_Q$$

- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics I!

Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \quad \stackrel{m_Q \to \infty}{\longrightarrow} \quad 0$$

  - Spin Rotations become a symmetry
  - Heavy Quark Spin Symmetry: SU(2) Rotations
- Spin Flavour Symmetry Multiplets

Mesonic Ground States

Bottom:

$$|(b\bar{u})_{J=0}\rangle = |B^-\rangle \qquad |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle$$

$$|(b\bar{d})_{J=0}\rangle = |\overline{B}^0\rangle \qquad |(b\bar{d})_{J=1}\rangle = |\overline{B}^{*0}\rangle$$

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Baryonic Ground States

$$\left|[(ud)_0 Q]_{1/2}\right\rangle = |\Lambda_Q\rangle$$

$$\left|[(uu)_1 Q]_{1/2}\right\rangle, \left|[(ud)_1 Q]_{1/2}\right\rangle, \left|[(dd)_1 Q]_{1/2}\right\rangle = |\Sigma_Q\rangle$$

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Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

$$\left\langle H^{(*)}(v)\right| Q_v \Gamma Q_{v'} \left| H^{(*)}(v')\right\rangle = C_\Gamma(v, v')\xi(v \cdot v')$$

with $H^{(*)}(v) = D^{(*)}(v)$ or $B^{(*)}(v)$

  - $C_\Gamma(v, v')$: Computable Clebsh Gordan Coefficient
  - $\xi(v \cdot v')$: Reduced Matrix Element
  - $\xi(v \cdot v')$: universal non-perturbative Form Factor: Isgur Wise Funktion
  - Normalization of ξ at $v = v'$:

$$\xi(v \cdot v' = 1) = 1$$

Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m_Q
- Various ways of derivaton
  - Non-relativistic reduction of the Dirac Equation: Foldy Wuithoisen Transformation
  - Integrating out heavy degrees of freedom
- At tree level, all this can be done explicitely

HQET from QCD

- Start from QCD: Action for a heavy quark Q

$$S = \int d^4x \, \bar{Q}(i\not{D} - m_Q)Q \qquad D_\mu = \partial_\mu + igA_\mu$$

- Generating functional for Greens functions

$$Z(\eta, \bar{\eta}, \lambda) = \int [dQ][d\bar{Q}][d\phi_\lambda]$$
$$\times \exp\left\{ iS + iS_\lambda + i \int d^4x \, (\bar{\eta}Q + \bar{Q}\eta + \phi_\lambda \lambda) \right\}$$

- $\phi_\lambda = q$, A_μ^a: light quark and gluon fields
- S_λ Action for the light d.o.f.

- How do we identify the heavy degrees of freedom?
- Slightly different situation compared to the previous examples: The number of heavy quarks is conserved

$\longrightarrow$ The heavy quark does not disappear from the theory

$\longrightarrow$ It remains in the theory as a static source of a color field

$\longrightarrow$ ... moving with the velocity of the heavy hadron

- Introduce a (four) velocity vector v with $v^2 = 1$, $v_0 > 0$

- Split into an “upper” and a “lower” component

$$\phi_v = \frac{1}{2}(1+\not{v})Q, \qquad \not{v}\phi_v = \phi,$$
$$\chi_v = \frac{1}{2}(1-\not{v})Q, \qquad \not{v}\chi_v = -\chi,$$

- Split the covariant derivative into a “temporal” and “spatial” part

$$D_\mu = v_\mu(v\cdot D) + D_\mu^\perp\,, \quad D_\mu^\perp = (g_{\mu\nu} - v_\mu v_\nu)D^\nu, \quad \{\not{D}^\perp\,,\,\not{v}\} = 0.$$

$$\boxed{S = \int d^4x \left[\bar{\phi}\{i(v\cdot D) - m_Q\}\phi - \bar{\chi}\{i(v\cdot D) + m_Q\}\chi + \bar{\phi}i\not{D}^\perp\chi + \bar{\chi}i\not{D}^\perp\phi\right]}$$

- Split the four momentum of the heavy quark:

$$p = m_Q v + k \,, \quad k : \text{residual momentum}$$

- This corresponds to a field redefinition:

$$\phi_v(x) = e^{-im_Q(v\cdot x)} h_v(x) \,, \qquad \chi_v(x) = e^{-im_Q(v\cdot x)} H_v(x)$$

with $i\partial h_v(x) \sim k$

- Insert this

$$S = \int d^4x \, \left[\bar{h}_v i(v\cdot D) h_v - \bar{H}_v \{i(v\cdot D) + 2m_Q\} H_v + \bar{h}_v i\not{D}^\perp H_v + \bar{H}_v i\not{D}^\perp h_v\right]$$

- The sources become

$$\int d^4x\,(\bar\eta\psi + \bar\psi\eta) = \int d^4x\,(\bar\rho_v h_v + \bar h_v \rho_v + \bar R_v H_v + \bar H_v R_v)$$

$\to$ Source terms for h_v and H_v

- Result for the generating function:

$$Z(\rho_v, \bar\rho_v, R_v, \bar R_v, \lambda) = \int [dh_v][d\bar h_v][dH_v][d\bar H_v][d\phi_\lambda]$$
$$\times \exp\left\{ iS + S_\lambda + i\int d^4x\,(\bar\rho_v h_v + \bar h_v \rho_v + \bar R_v H_v + \bar H_v R_v + \phi_\lambda \lambda)\right\}$$

- Interpretation: H_v is “heavy” with mass $2m_Q$
  h_v is “light”, massless
- Integrate out the field H_v, with $R_v = \bar R_v = 0$

This can be done explicitely: “Gaussian integration”

$$Z(\rho_v, \bar{\rho}_v, \lambda) = \int [dh_v][d\bar{h}_v][d\lambda]\, \Delta \times \exp\left\{ iS + S_\lambda + i \int d^4x \, (\bar{\rho}_v^+ h_v^+ + \bar{h}_v^+ \rho_v^+ + \phi_\lambda \lambda) \right\}$$

$$S = \int d^4x \left[\bar{h}_v^+ i(v \cdot D) h_v^+ - \bar{h}_v^+ \not{D}^\perp \left(\frac{1}{i(v \cdot D) + 2m_Q - i\epsilon} \right) \not{D}^\perp h_v^+ \right]$$

This is a nonlocal functional

$$\Delta = \text{Determinant of} \quad (i(v \cdot D) + 2m_Q - i\epsilon)$$

Can be shown to be a constant

HQET Lagrangian

- Expand S in local terms: HQET Lagrangian

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \frac{1}{2m_Q}\sum_{n=0}^{\infty}\bar{h}_v(i\not{D}_\perp)\left(\frac{ivD}{2m_Q}\right)^n(i\not{D}_\perp)h_v$$

$$= \bar{h}_v(ivD)h_v \qquad \text{Dimension 4}$$

$$+\frac{1}{2m_Q}\bar{h}_v(i\not{D}_\perp)^2h_v \qquad \text{Dimension 5}$$

$$+\left(\frac{1}{2m}\right)^2\bar{h}_v(i\not{D}_\perp)(-ivD)(i\not{D}_\perp)h_v \qquad \text{Dimension 6}$$

$$+\cdots$$

- Expansion of the heavy quark field:
  (Relevant once we consider a weak current)

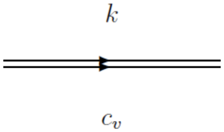
$$Q(x) = e^{-im_Q vx}\left[1 + \frac{1}{2m}\sum_{n=0}^{\infty}\left(\frac{ivD}{2m_Q}\right)^n i\not{D}_\perp\right] h_v$$
$$= e^{-im_Q vx}\left[1 + \frac{1}{2m_Q}\not{D}_\perp + \left(\frac{1}{2m_Q}\right)^2(-ivD)\not{D}_\perp + \cdots\right] h_v$$

- This is the whole story at tree level ...

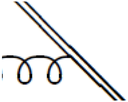
HQET Loops

Feynman Rules of HQET: Read off from the dim-4 piece:

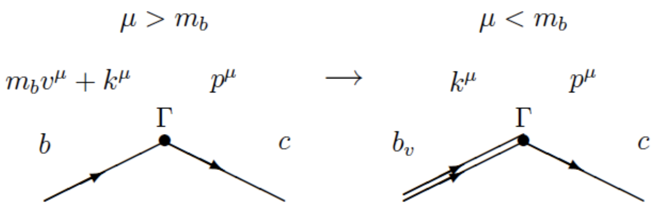
- Propagator:



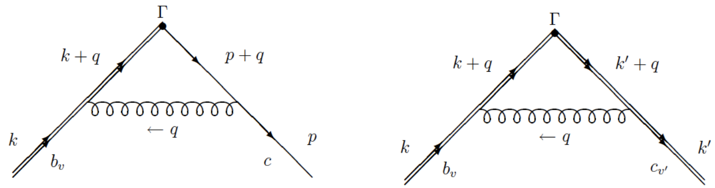
$$\longrightarrow \quad \frac{i}{v \cdot k + i\epsilon}$$

- Vertex:  $\longrightarrow \quad (-i) g_s T^a v^\mu$

e.g. Weak $b \to c$ current



@ one loop



Calculate the anomalous dimensions of the diagrams:

- $\mu \geq m_b$: Full QCD, no anomalous dimension (due to the conservation of the left haned current in the massless limit)
- $m_c \leq \mu \leq m_b$: $\boxed{\gamma_{\text{HL}} = -\frac{\alpha_s}{\pi}}$, independent of Γ
- $m_c \leq \mu \leq m_b$:

$$\gamma_{\text{HH}} = \frac{4\alpha_s}{3\pi}\left[(v \cdot v')r(v \cdot v') - 1\right] \quad \text{with}$$

$$r(w) = \frac{1}{\sqrt{w^2 - 1}} \ln\left[w + \sqrt{w^2 - 1}\right]$$

Depends on the $(v \cdot v')$, which is an external parameter

Remarks:

- Running from M_W to m_b: Resummation of $\ln\left[\frac{M_W^2}{m_b^2}\right]$

  There are no logs like this for the left handed current!
- Running from m_b to m_c: Resummation of $\ln\left[\frac{m_b^2}{m_c^2}\right]$
- Running from m_c to Λ: Resummation of $\ln\left[\frac{m_c^2}{\Lambda^2}\right]$
- The naive calculation (with all masses non-vanishing) contains all these logs at order α_s
- Renorm. Group resumms these logs

Matching Coefficients at $v = v'$

- Vector Current

$$\eta_V = 1 + \frac{\alpha_s(\mu)}{\pi} \left[\frac{m_b + m_c}{m_b - m_c} \ln\left(\frac{m_b}{m_c}\right) - 2 \right] = 0.98$$

- Axial Vector Current

$$\eta_V = 1 + \frac{\alpha_s(\mu)}{\pi} \left[\frac{m_b + m_c}{m_b - m_c} \ln\left(\frac{m_b}{m_c}\right) - \frac{8}{3} \right] = 1.02$$

for $\mu \sim m_b$

More on Heavy Quark Symmetries: Luke Theorem

... a nice app. of Heavy Flavour Symmetry: $SU(2)$

$$Q_+ = \int d^3x\, \bar{b}_v(x) c_v(x) \,, \quad Q_- = \int d^3x\, \bar{c}_v(x) b_v(x) \,,$$

$$Q_3 = \int d^3x\, (\bar{b}_v(x) b_v(x) - \bar{c}_v(x) c_v(x)) \,,$$

$$[Q_+, Q_-] = Q_3 \,, \qquad [Q_+, Q_3] = -2Q_+ \,, \qquad (Q_+)^\dagger = Q_-$$

- $SU(2)$ Commutation relations
- Ground state doublet: $|B\rangle$ and $|D\rangle$ (with equal velocities)

$$\begin{aligned} H &= H_0^{(b)} + H_0^{(c)} + \frac{1}{m_b} H_1^{(b)} + \frac{1}{m_c} H_1^{(c)} + \cdots \\ &= H_0^{(b)} + H_0^{(c)} + \frac{1}{2}\left(\frac{1}{m_b} + \frac{1}{m_c}\right)(H_1^{(b)} + H_1^{(c)}) \\ &\qquad + \frac{1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)(H_1^{(b)} - H_1^{(c)}) + \cdots \\ &= H_{symm} + H_{break} \end{aligned}$$

- The last term does not commute with $Q_\pm$, but still commutes with Q_3
- Still common eigenstates of H_{symm} and Q_3: $|\tilde{B}\rangle$ and $|\tilde{D}\rangle$

$$
\begin{aligned}
1 &= \langle \tilde{B} | Q_3 | \tilde{B} \rangle = \langle \tilde{B} | [Q_+, Q_-] | \tilde{B} \rangle \\
&= \sum_n \left[\langle \tilde{B} | Q_+ | \tilde{n} \rangle \langle \tilde{n} | Q_- | \tilde{B} \rangle - \langle \tilde{B} | Q_- | \tilde{n} \rangle \langle \tilde{n} | Q_+ | \tilde{B} \rangle \right] \\
&= \sum_n \left[|\langle \tilde{B} | Q_+ | \tilde{n} \rangle|^2 - |\langle \tilde{B} | Q_- | \tilde{n} \rangle|^2 \right]
\end{aligned}
$$

- $|\tilde{n}\rangle$: Complete set of eigenstates of $H_{symm} + H_{break}$, hence

$$
\langle \tilde{B} | Q_\pm | \tilde{n} \rangle = \frac{1}{E_B - E_n} \langle \tilde{B} | [H_{break}, Q_\pm] | \tilde{n} \rangle \quad \text{since} \quad [H_{symm}, Q_\pm] = 0
$$

(a) In case $|\tilde{n}\rangle = |\tilde{D}\rangle$:
$\langle \tilde{B}|Q_\pm|\tilde{D}\rangle \sim \mathcal{O}(1)$ and hence $E_B - E_n \sim \mathcal{O}(H_{break})$

(b) In case $|\tilde{n}\rangle \neq |\tilde{D}\rangle$:
$E_B - E_n \sim \mathcal{O}(1)$ and hence $\langle \tilde{B}|Q_\pm|\tilde{n}\rangle \sim \mathcal{O}(H_{break})$

From this we get

$$\boxed{\langle \tilde{B}|Q_+|\tilde{D}\rangle = 1 + \mathcal{O}\left[\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)^2\right]}$$

Ademollo Gatto Theorem, Luke's theorem
Corrections to the Wigner Eckart Theorem!

Sample Application: $B \longrightarrow D^{(*)}\ell\bar{\nu}_\ell$

This is the obvious playground:

- This is a heavy-to-heavy transition:
  Governed by the Wigner Eckart Theorem of HQS
- The weak current is a generator of HQS:
  The absolute Normalization of the Form Factor is known
- The Ademollo Gatto Theorem / Luke's Thoerem applies:
  Corrections to certain form factor normalziations are of second order

Determination of V_{cb} from $B \to D^{(*)}\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3(\omega^2-1)^{1/2}P(\omega)(\mathcal{F}(\omega))^2$$
$$\frac{d\Gamma}{d\omega}(B \to D\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3(\omega^2-1)^{3/2}(\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors, expressible in terms of $\xi(\omega)$

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] + (\omega - 1)\rho^2 + \mathcal{O}((\omega-1)^2)$$

$$\mathcal{G}(1) = \eta_{\rm QED}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\rm QED} = 1.007$

$B \longrightarrow D^{(*)}$ Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.927 \pm 0.024}$$

$$\boxed{\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016}$$

F(1): Milc/Fermilab 2009, G(1): A. Kronfeld et al. 2005

$B \longrightarrow D^{(*)}$ Form Factors: Non-Lattice Results

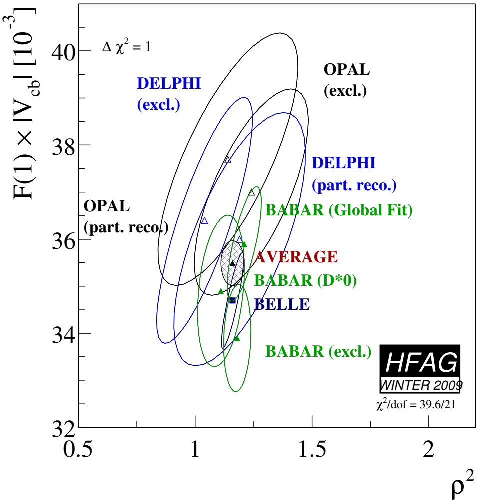
- $B \longrightarrow D^*$ Form Factor:
  - Based on Zero Recoil Sum Rules (Uraltsev, also Ligeti et al.)
  - Including full α_s and up to $1/m_b^5$

$$\boxed{\mathcal{F}(1) = 0.86 \pm 0.04}$$ (Gambino, Uraltsev, M (2010))

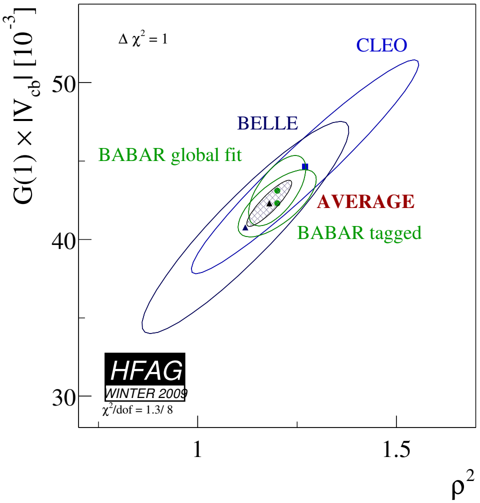
- $B \longrightarrow D$ Form Factor:
  - Based on the “BPS limit” $\mu_\pi^2 = \mu_G^2$

$$\boxed{\mathcal{G}(1) = 1.04 \pm 0.02}$$ (Uraltsev)

$B \longrightarrow D^* \ell \bar{\nu}_\ell$



$B \longrightarrow D \ell \bar{\nu}_\ell$



$$\boxed{V_{cb,excl} = (38.7 \pm 1.1) \times 10^{-3}}$$

(Lattice)

$$\boxed{V_{cb,excl} = (41.0 \pm 1.5) \times 10^{-3}}$$

(ZR Sum Rules. prelim.)

Currently under Debate ....

Summary of Part II

- Heavy Mass limit as an effective Field Theory:
  Expansion in inverse powers of the heavy mass
- Perturbative calculations in HQET: Anomalous dimensions
- Key ingredient for phenomenology:
  Heavy Quark Symmetries $\rightarrow$ Relations between nonperturbative matrix elements
- This has been the end of the era of form factor models ... (for most applications)